PRACTICE TEST 4 EXPLANATIONS

Section I: Multiple-Choice Questions

1. D

The total distance traveled is equal to the sum of the individual distances traveled by the explorer: \( d = 30 \text{ m} + 20\sqrt{2} \text{ m} + 140 \text{ m} = 198.2 \text{ m} \).

2. A

The displacement is equal to the change in position of the explorer. The horizontal and vertical components of the explorer’s displacement can be calculated as follows:

\[
\Delta x = 30 \text{ m} + 20\sqrt{2} \cos45^\circ = 50 \text{ m}
\]

\[
\Delta y = -20\sqrt{2} \sin45^\circ + 140 \text{ m} = 120 \text{ m}
\]

The displacement of the explorer then is the magnitude of the vector \((50i + 120j) \text{ m}):\

\[
\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(50 \text{ m})^2 + (120 \text{ m})^2} = 130 \text{ m}
\]

3. A

As the displacement of the explorer was 130 m, the average velocity of the explorer is equal to

\[
\text{average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{130 \text{ m}}{260 \text{ s}} = 0.5 \text{ m/s}
\]

4. D

The position of the object is decreasing so the object must be moving in the negative direction with a negative velocity. Eliminate (A) and (B). The position of the object is decreasing at a slower rate over time, so the magnitude of the velocity must be decreasing. Eliminate (C). Choice (D) is correct.

5. D

In section 1, the object is slowing down and moving in the positive direction. Eliminate (A). In section 2, the object is speeding up and moving in the negative direction. Eliminate (B). In section 3, the object is moving with constant speed in the negative direction. Eliminate (C). In section 4, the object is slowing down and moving in the negative direction. Choice (D) is correct.

6. A

When a projectile reaches the top of its trajectory, the vertical component of its velocity is momentarily zero. As the horizontal velocity in standard
parabolic motion is always constant, this means that the velocity of projectiles is smallest at the apex of the trajectory. Choice (A) is correct. Choice (B) is incorrect as the projectile can still have a horizontal velocity at the apex. The acceleration experienced by projectiles in flight is gravity, which is constant at all points of the trajectory. Eliminate (C). Projectiles have the smallest velocity at the apex of the trajectory, so they cannot have the maximum kinetic energy at this same point. Eliminate (D).

7. **C**

First, calculate how long it takes for the cannonball to reach the wall. This is a horizontal question. As the horizontal velocity is constant, the time it takes for the cannonball to reach the wall can be calculated as follows:

\[
\Delta x = v_{0x}t \implies t = \frac{\Delta x}{v_{0x}} = \frac{30 \text{ m}}{20 \cos 45^\circ} = 2.12 \text{ s}
\]

Next, calculate the height of the projectile at this time by applying Big Five #3:

\[
\Delta y = v_{0y}t - \frac{1}{2}gt^2 = 20 \sin 45^\circ (2.12 \text{ s}) - \frac{1}{2}(10 \text{ m/s}^2)(2.12 \text{ s})^2 = 7.51 \text{ m}
\]

This is the maximum height of a wall that the cannonball can clear.

8. **B**

Use Big Five #1:

\[
\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(0 \text{ m/s} + 15 \text{ m/s})(5 \text{ s}) = 37.5 \text{ m}
\]

9. **B**

The average velocity is given by the equation:

\[
\bar{v} = \frac{\Delta x}{\Delta t} = \frac{200 \text{ m} - 40 \text{ m}}{5 \text{ s}} = 32 \text{ m/s}
\]

10. **C**

In the horizontal direction, there are two forces acting on the book: the force applied by the student and the normal force from the wall. In the vertical direction, there are also two forces acting on the book: the force of friction and the weight of the book. In order for the student to hold the book in place, the net force on the book must equal zero in both the horizontal and vertical directions.

\[
\sum F_x = F_N - F_{app} = 0 \implies F_N = F_{app}
\]

\[
\sum F_y = F_I - F_g = 0 \implies F_I \implies F_g = \mu F_N = \frac{mg}{\mu_s} = 0.5 \text{ kg} \left(10 \text{ m/s}^2\right) = 25 \text{ N}
\]
As the normal force is equal to the force applied, the student must apply a force of 25 N to hold the book in place.

11. D

There are four forces acting on the box: the weight of the box, the normal force of the inclined plane on the box, the force applied, and the force of friction. The weight of the box can be separated into its parallel and perpendicular components.

In order for the box to be at rest, the net force in both the parallel and perpendicular directions must be zero:

\[ F_N = F_w \cos(\theta) \]
\[ F_N = mg \cos(\theta) \]
\[ F_w \sin(\theta) - F_f - F_{\text{app}} = 0 \]
\[ F_{\text{app}} = F_w \sin(\theta) - F_f = mg \sin(\theta) - \mu F_N \]
\[ = (10 \text{ kg})(10 \text{ m/s}^2)(\sin 60^\circ) - 0.2(10 \text{ kg})(10 \text{ m/s}^2)(\cos 60^\circ) = 76.6 \text{ N} \]

A force of 76.6 N must be applied to prevent the box from sliding down the inclined plane.

12. C

By Newton's Third Law, when the heavier car exerts a force on the smaller car, that small car exerts an equal but opposite force back onto the heavy car. While the forces are in opposite direction, both forces have the same magnitude.

13. D
By Newton’s Second Law, the acceleration of each mass can be evaluated using \( F = ma \). As both objects experience the same magnitude of force, the \( a_1/a_2 \) can be evaluated as follows:

\[
F_1 = F_2 \\
m_1a_1 = m_2a_2 \\
\frac{a_1}{a_2} = \frac{m_2}{m_1} = \frac{40 \text{ kg}}{10 \text{ kg}} = 4
\]

14. **D**

There are two forces acting on the box: the weight of the box and the tension force from the string. Newton’s Second Law gives the acceleration of the box:

\[
F_{\text{net}} = ma = F_T - F_g
\]

The maximum acceleration of the block is limited by the breaking strength of the string. The maximum acceleration that the box can have is

\[
ma = F_T - F_T \Rightarrow a = \frac{F_T - F_T}{m} = \frac{80 \text{ N} - 4 \text{ kg}(10 \text{ m/s}^2)}{4 \text{ kg}} = 10 \text{ m/s}^2
\]

15. **C**

The work done can be calculated as follows:

\[
W = Fd \cos \theta = 40 \text{ N} \cdot 20 \text{ m} \cdot \cos 60^\circ = 400 \text{ J}
\]

16. **C**

Apply Conservation of Mechanical Energy:

\[
K_i + U_i = K_f + U_f \\
o + mgh = \frac{1}{2}mv_f^2 + 0
\]

\[
v_f = 2gh = \sqrt{2gh} = \sqrt{2(10 \text{ m/s}^2)(20 \text{ m})} = 20 \text{ m/s}
\]

17. **D**

The work done can be calculated using the Work–Energy Theorem:

\[
W = \Delta K = K_{\text{final}} - K_{\text{initial}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m(v^2 - v_0^2)
\]

\[
= \frac{1}{2}(0.2 \text{ kg})(30 \text{ m/s}^2 - 0) = 90 \text{ J}
\]
18. C

Initially, all of the energy of the block is potential energy. At the halfway point of the block, half of the potential energy of the block has been converted to kinetic energy. However, the force of friction does negative work on the block, which reduces the kinetic energy of the block. As a result, the kinetic energy of the block at the halfway point is equal to

\[ K = \frac{mgh}{2} = W_f \]

The length of the inclined plane can be calculated as follows: \( \sin \theta = \frac{2 \text{ m}}{d} \Rightarrow d = \frac{2 \text{ m}}{\sin 30^\circ} = 4 \text{ m} \). As the block is halfway down, the block has traveled a distance of 2 m.

\[
K = \frac{mgh}{2} - F_fd = \frac{mgh}{2} - \mu_km \cdot \cos \theta
\]

\[
= \frac{10 \text{ kg}(10 \text{ m/s}^2)(2 \text{ m})}{2} - 0.1(10 \text{ kg})(10 \text{ m/s}^2)(\cos 30^\circ) = 91 \text{ J}
\]

19. C

The power can be calculated as follows:

\[ P = Fv = 2,000 \text{ N}(30 \text{ m/s}) = 60,000 \text{ W} \]

20. D

The impulse is equal to the change in linear momentum, which can be calculated as follows:

\[ J = \Delta p = m\Delta v = m(v_f - v_i) = 2 \text{ kg}(10 \text{ m/s} - (-20 \text{ m/s})) = 60 \text{ N} \cdot \text{s} \]

21. C

The average force experienced by the ball can be calculated using

\[ F = \frac{\Delta p}{\Delta t} = \frac{60 \text{ N} \cdot \text{s}}{0.002 \text{ s}} = 30,000 \text{ N} \]

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22. **A**

The work done can be calculated using the Work–Energy Theorem:

\[
W = \Delta K = K_{\text{final}} - K_{\text{initial}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \frac{1}{2}m(v^2 - v_o^2)
\]

\[
= \frac{1}{2}(2 \text{ kg})[(10 \text{ m/s})^2 - (20 \text{ m/s})^2] = -300 \text{ J}
\]

23. **B**

As gravity is a conservative force, the work done by gravity is path independent and can be calculated as

\[
\Delta U_g = -W_g \Rightarrow W_g = -\Delta U_g = -mg(h_f - h_i) = -6 \text{ kg}(10 \text{ m/s}^2)(10 \text{ m} - 0 \text{ m}) = -600 \text{ J}
\]

24. **C**

The objects did not stick together after the collision, so (B) is wrong. To determine whether the collision is elastic or inelastic, calculate the total initial and final kinetic energies:

\[
K_i = \frac{1}{2}m_1v_{1,i} = \frac{1}{2}(2 \text{ kg})(6 \text{ m/s})^2 = 36 \text{ J}
\]

\[
K_f = \frac{1}{2}m_1v_{1,f} + \frac{1}{2}m_2v_{2,f} = \frac{1}{2}(2 \text{ kg})(2 \text{ m/s})^2 + \frac{1}{2}(1 \text{ kg})(8 \text{ m/s})^2 = 36 \text{ J}
\]

As kinetic energy was conserved, the collision was elastic.

25. **B**

The final velocity of a perfectly inelastic collision can be calculated using the equation:

\[
m_1v_{1,i} + m_2v_{2,i} = (m_1 + m_2)v_f
\]

\[
v_f = \frac{m_1v_{1,i} + m_2v_{2,i}}{m_1 + m_2} = \frac{3 \text{ kg}(3 \text{ m/s}) + 2 \text{ kg}(-6 \text{ m/s})}{3 \text{ kg} + 2 \text{ kg}} = -0.6 \text{ m/s}
\]

26. **C**

As the cannon and cannonball are both initially at rest, the total initial momentum is 0. By Conservation of Linear Momentum, the total final momentum must also be equal to 0.

\[
m_{\text{cannon}}v_{\text{cannon},i} + m_{\text{cannonball}}v_{\text{cannonball},i} = m_{\text{cannon}}v_{\text{cannon},f} + m_{\text{cannonball}}v_{\text{cannonball},f}
\]

\[
0 = m_{\text{cannon}}v_{\text{cannon},f} + m_{\text{cannonball}}v_{\text{cannonball},f}
\]
27. **D**

When the car is making the curve, it is the force of friction providing the centripetal force:

\[
F_c = F_f \\
\frac{mv^2}{r} = \mu F_N \\
\frac{mv^2}{r} = \mu mg \\
\frac{v^2}{r} = \mu g \\
v^2 = r\mu g \\
v = \sqrt{r\mu g} = \sqrt{50 \text{ m}(0.8)(10 \text{ m/s}^2)} = 20 \text{ m/s}
\]

28. **C**

The gravitational pull of the planet provides the centripetal force on the satellite:

\[
F_g = F_c \\
G \frac{Mm}{r^2} = \frac{mv^2}{r} \Rightarrow G \frac{M}{r} = v^2 \Rightarrow v = \sqrt{\frac{GM}{r}}
\]

The speed of the second satellite is

\[
v_2 = \sqrt{\frac{GM}{2r}} = \frac{1}{\sqrt{2}} \sqrt{\frac{GM}{r}} = \frac{1}{\sqrt{2}} v
\]

29. **D**

The reading on the scale is equal to the magnitude of the downward force exerted by the man on the scale. By Newton’s Third Law, this is equivalent to the magnitude of the normal force exerted by the scale on the man. There are two forces acting on the man: the weight of the man and the normal force of the scale. Apply Newton’s Second Law:
\[ F_{\text{net}} = F_N - F_g = ma \]
\[
F_N = ma + F_r = \frac{mg}{4} + mg = \frac{5mg}{4} = \frac{5(50 \text{ kg})(10 \text{ m/s}^2)}{4} = 625 \text{ N}
\]

30. **C**

Apply Big Five #2 for rotational motion:
\[
\omega = \omega_0 + \alpha t = 0 + 2 \text{ rad/s}\cdot(5 \text{ s}) = 10 \text{ rad/s}
\]

The angular velocity can be related to linear velocity with the equation:
\[
v = r\omega = (0.5 \text{ m})(10 \text{ rad/s}) = 5 \text{ m/s}
\]

31. **A**

Start by calculating the center of mass for the system with two masses:
\[
x(\text{c.m.}) = \frac{x_1m_1 + x_2m_2}{m_1 + m_2} = \frac{0 \text{ m}(3 \text{ kg}) + 1 \text{ m}(5 \text{ kg})}{3 \text{ kg} + 5 \text{ kg}} = 0.625 \text{ m}
\]

Next, calculate the new center of the mass of the system with the addition of the third mass:
\[
x(\text{c.m.}) = \frac{x_1m_1 + x_2m_2 + x_3m_3}{m_1 + m_2 + m_3} = \frac{0 \text{ m}(3 \text{ kg}) + 1 \text{ m}(5 \text{ kg}) + 0.5 \text{ m}(2 \text{ kg})}{3 \text{ kg} + 5 \text{ kg} + 2 \text{ kg}} = 0.6 \text{ m}
\]

The center of mass of the system has shifted to the left by 0.025 m.

32. **B**

The weight of the box pulls down on the rope, producing tension that creates a torque on the pulley. This torque is equal to
\[
\tau = rF = rF_g = rmg = 0.5 \text{ m}(5 \text{ kg})(10 \text{ m/s}^2) = 25 \text{ N}\cdot\text{m}
\]

33. **D**

Rotational inertia is directly proportional to the mass of the object. Eliminate (A) and (B), as they have smaller masses. The farther away the mass is from the axis of rotation, the greater the rotational inertia. The mass of a hollow ball is farther away from the axis of rotation than that of a solid ball. Eliminate (C). Choice (D) is correct.

34. **C**

Apply Hooke’s Law to calculate the force constant of the spring:
\[
F = kx \Rightarrow k = \frac{F}{x} = \frac{6 \text{ N}}{0.05 \text{ m}} = 120 \text{ N/m}
\]
The frequency of a spring-block oscillator is given by

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{120 \text{ N/m}}{5 \text{ kg}}} = 0.78 \text{ Hz} \]

35. C

When the block is at its amplitude, all of the kinetic energy has been converted to potential energy. The maximum potential energy is given by \( U = \frac{1}{2} kA^2 \). The amplitude can then be calculated by applying Conservation of Energy:

\[ K_{\text{max}} = U_{\text{max}} \]

\[ \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} kA^2 \]

\[ m v_{\text{max}}^2 = kA^2 \]

\[ A^2 = \frac{m v_{\text{max}}^2}{k} \]

\[ A = \sqrt{\frac{m v_{\text{max}}^2}{k}} = \sqrt{\frac{3 \text{ kg}(4 \text{ m/s})^2}{10 \text{ N/m}}} = 2.2 \text{ m} \]

36. C

The period of a spring is given by

\[ T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow \sqrt{\frac{L}{g}} = \frac{T}{2\pi} \Rightarrow \frac{L}{g} = \left(\frac{T}{2\pi}\right)^2 \Rightarrow L = g \left(\frac{T}{2\pi}\right)^2 = 10 \frac{\text{m}}{\text{s}^2} \left(\frac{2 \text{ s}}{2\pi}\right)^2 = 1 \text{ m} \]

37. B

The frequency can be calculated using the following equation:

\[ \nu = \lambda f \Rightarrow f = \frac{\nu}{\lambda} = \frac{343 \text{ m/s}}{20 \times 10^{-2} \text{ m}} = 1,715 \text{ Hz} \]

38. D

Standing waves have zero amplitude at nodes and maximum amplitude at antinodes. Eliminate (A) and (B). In standing waves, complete constructive interference occurs at antinodes and complete destructive interference occurs at nodes. Eliminate (C). Choice (D) is correct.

39. C
The length of the string is equal to 2.5\( \lambda \), so

\[ L = 2.5\lambda \Rightarrow \lambda = \frac{L}{2.5} = \frac{2L}{5} \]

The expression for the wavelength of the nth harmonic is \( \frac{2L}{n} \), so the harmonic number of the standing wave is 5.

40. A

The frequency of a standing wave in an open tube is given by the following equation:

\[ f_n = \frac{nv}{4L} \]

The frequency of the third harmonic is

\[ f_3 = \frac{3(343 \text{ m/s})}{4(0.5 \text{ m})} = 514.5 \text{ Hz} \]

41. D

The electric force between two particles with charges of \( q_1 \) and \( q_2 \), separated by a distance \( r \), is given by the following equation:

\[ F_E = k \frac{q_1 q_2}{r^2} \]

If the magnitude of each charge is halved and the distance between their centers is halved, then the electric force will be

\[ F'_E = k \frac{(0.5q_1)(0.5q_2)}{(0.5r)^2} = k \frac{0.25q_1 q_2}{0.25r^2} = k \frac{q_1 q_2}{r^2} \]

The magnitude of the electric force remains the same.

42. A

As the train approaches the person, the person perceives the sound of the horn with a frequency higher than the original frequency emitted from the train due to the Doppler effect. As the train slows, the person still perceives a higher frequency but it is not as high as when the train was traveling faster. As a result, the person hears a decrease in the frequency of the horn. As the train approaches, the distance between the person and the train decreases. This leads to an increase in the intensity of the horn perceived by the person.

43. B
Choice (A) would be an example of the Doppler effect. Choices (C) and (D) would both affect the speed of the wave, which would change the wavelength because $v = f \lambda$; the frequency of the wave would not change, meaning that the wavelength would change proportionally to the speed. Choice (B) is correct because pressure has no effect on the speed of sound. If the speed and frequency both remain constant, then the wavelength must as well.

44. D

The equivalent resistance of $R_2$ and $R_3$ is

$$R_{2+3} = R_2 + R_3 = 1 \, \Omega + 2 \, \Omega = 3 \, \Omega$$

The equivalent resistance of $R_{2+3}$ and $R_1$ is

$$\frac{1}{R_{1+2+3}} = \frac{1}{R_1} + \frac{1}{R_{2+3}}$$

$$\frac{1}{R_{1+2+3}} = \frac{1}{6 \, \Omega} + \frac{1}{3 \, \Omega} = \frac{1}{2 \, \Omega} \Rightarrow R_{1+2+3} = 2 \, \Omega$$

The total resistance of the circuit then is

$$R_{\text{Tot}} = R_{1+2+3} + R_4 = 2 \, \Omega + 4 \, \Omega = 6 \, \Omega$$

45. C

The power of the circuit is equal to

$$P = \frac{V^2}{R} = \frac{(10 \, \text{V})^2}{6 \, \Omega} = \frac{50}{3} \, \text{W}$$

The amount of time it would take for the battery to deliver 300 J of energy is equal to

$$\frac{300 \, \text{J}}{50 / 3 \, \text{W}} = 18 \, \text{s}$$

46. B, D

A parabolic position-versus-time graph is indicative of an object with a constant magnitude of acceleration. Eliminate (C). The position of the object is decreasing so the object is moving in the negative direction. Eliminate (A). The position of the object is decreasing at a slower rate over time, so the speed of the object must be decreasing. Choices (B) and (D) are correct.

47. C, D
According to the Wave Rule #2, when a wave passes into another medium, its speed changes, but its frequency does not. By the equation \( v = \lambda f \), the wavelength of the wave must also change.

48.  **B, D**

Choices (B) and (D) are both true statements regarding objects undergoing uniform circular motion. As the objects are moving in a circle, the direction of their velocity is constantly changing. Choice (A) is wrong. The centripetal acceleration is directed toward the center of the circle. Choice (C) is wrong.

49.  **B, C**

The work done by conservative forces is path independent. Gravity is an example of a conservative force. The work done by nonconservative forces is path dependent. Choice (A) is wrong. Friction is a nonconservative force. Choice (D) is wrong.

50.  **C, D**

According to Wave Rule #1, the speed of a wave is determined by the type of wave and the characteristics of the medium. The speed of a wave does not depend on the energy or frequency of the wave. Choices (A) and (B) are wrong.

**Section II: Free-Response Questions**

1. (a) In a perfectly elastic collision, both momentum and kinetic energy are conserved:

   Equation 1: \( m_1v = m_1v_1 + m_2v_2 \)

   Equation 2: 
   \[
   \frac{1}{2} m_1v^2 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2 \Rightarrow m_1v^2 = m_1v_1^2 + m_2v_2^2
   \]

   Rearrange these equations into

   \[
   m_1v = m_1v_1 + m_2v_2 \Rightarrow m_2v_2 = m_1(v - v_1)
   \]

   \[
   m_2v_2^2 = m_1v^2 - m_1v_1^2 \Rightarrow m_2v_2^2 = m_1(v - v_1)(v + v_1)
   \]

   Divide the second rearranged equation by the first rearranged equation to get

   \[
   v_2 = v + v_1
   \]

   Substitute this value back into Equation 1 to get

   \[
   m_1v = m_1v_1 + m_2v_2
   \]

   \[
   m_1v = m_1v_1 + m_2v + m_2v_1
   \]

   \[
   m_1v_1 + m_2v_1 = m_1v - m_2v
   \]

   \[
   v_1 = \frac{m_1v - m_2v}{m_1 + m_2}
   \]
(b) Substitute the expression for \( v_1 \) from (a) back into \( v_2 = v + v_1 \):

\[
v_2 = v + \frac{m_1 v - m_2 v}{m_1 + m_2} = \frac{m_1 v + m_2 v}{m_1 + m_2} \]

(c) According to the expression for \( v_2 \) from (b), \( v_2 \) will always have a positive value. The two masses will be traveling in the same direction if \( v_1 \) is also positive. According to the expression for \( v_1 \) from (a), this is true if \( m_1 > m_2 \). If \( m_1 < m_2 \), the two masses will be traveling in the opposite direction. Note: if \( m_1 = m_2 \), the final velocity of \( m_1 \) would be zero, but \( m_2 \) would still be moving in the positive direction.

2. (a) The magnitude of the electric force is given by the following equation:

\[
F_E = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} = \left(9 \times 10^9 \, \text{N} \cdot \text{m}^2/\text{C}^2\right) \frac{3 \times 10^{-9} \, \text{nC} \times (-12 \times 10^{-9} \, \text{nC})}{(0.3 \, \text{m})^2} = -3.6 \times 10^{-6} \, \text{N}
\]

The electric force between two opposite charges is attractive.

(b) The electric field is given by the equation \( E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \). As there are two charges present, the electric field at any point is equal to the sum of the individual electric fields produced by each charge. As you are looking for the point where \( E = 0 \), set the total electric field equal to 0 and solve:

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_1^2} + \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_2^2} = 0
\]

\[
\frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_1^2} = -\frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_2^2}
\]

\[
\frac{q_1}{r_1^2} = -\frac{q_2}{r_2^2}
\]

\[
\frac{3 \, \text{nC}}{r_1^2} = -\frac{12 \, \text{nC}}{r_2^2}
\]

\[
\frac{1}{r_1^2} = \frac{4}{r_2^2}
\]

\[
4r_1^2 = r_2^2
\]

\[
2r_1 = r_2
\]

The electric field will be equal to 0 when \( 2r_1 = r_2 \). As you are looking for a point between the two charges, it is also true that
\[ r_1 + r_2 = 0.3 \text{ m} \]

Substituting in for \( r_2 \), you get
\[ r_1 + 2r_1 = 0.3 \text{ m} \Rightarrow 3r_1 = 0.3 \text{ m} \Rightarrow r_1 = 0.1 \text{ m} \]
\[ r_2 = 2r_1 = 0.2 \text{ m} \]

The electric field is equal to 0 at the point 0.1 m from the \( q_1 \) and 0.2 m from \( q_1 \) between the two charges.

(c) At the midpoint between the two charges, \( r_2 = r_1 = 0.15 \text{ m} \). The electric field at the point right between the two charges is equal to

\[
E = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} \right) = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1 + q_2}{r^2} \right)
\]
\[
= \left( 9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \left( \frac{3 \times 10^{-7} \text{ nC} + -12 \times 10^{-7} \text{ nC}}{(0.15 \text{ m})^2} \right) = -3.6 \times 10^3 \text{ N/C}
\]

As electric fields point away from positive charges and toward negative charges, the direction of the electric field is toward \( q_2 \).

(d) Given the electric field at a point, the electric force on a charge at that point is given by

\[ F = qE = (2 \times 10^{-9} \text{ nC})(-3.6 \times 10^3 \text{ N/C}) = -7.2 \times 10^{-6} \text{ N} \]

3. (a) When pulled to a distance of 0.5 m, the block has maximum potential energy given by the equation \( U = \frac{1}{2}kA^2 \). All of this energy is converted to kinetic energy when the block reaches maximum speed:

\[
K_{\text{max}} = U_{\text{max}}
\]
\[
\frac{1}{2}m v_{\text{max}}^2 = \frac{1}{2}kA^2
\]
\[
m v_{\text{max}}^2 = kA^2
\]
\[
v_{\text{max}} = \sqrt{\frac{kA^2}{m}} = A \sqrt{\frac{k}{m}} = 0.05 \text{ m} \left( \sqrt{\frac{40 \text{ N/m}}{0.1 \text{ kg}}} \right) = 1 \text{ m/s}
\]

(b) The frequency of oscillations is given by the following equation:

\[
f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{40 \text{ N/m}}{0.1 \text{ kg}}} = \frac{10}{\pi} \text{ Hz} = 3.2 \text{ Hz}
\]

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(c) When the spring is placed vertically on the ground with the block on top, there are two forces acting on the block at the new natural length: the weight of the block and the upwards restoring force. These two forces are equal to each other at the natural length:

\[ F_{\text{rest}} = F_g \]
\[ kx = mg \]

\[ x = \frac{mg}{k} = \frac{(0.1 \text{ kg})(10 \text{ m/s}^2)}{40 \text{ N/m}} = 0.025 \text{ m} \]

The natural length of the spring would decrease by 0.025 m.

(d) The frequency of spring-block oscillator is given by the equation \( f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \).

Flipping the spring vertically does not affect \( k \) or \( m \), so the frequency of oscillations will remain unchanged.

4. (a) The forces acting on the \( m \) mass are \( F_T \) (the tension in the string connecting it to \( M \) mass), \( F_w \) (the weight of the mass \( m \)), \( F_N \) (the normal force exerted by the inclined plane), and \( F_f \) (the force of static friction).

The forces acting on the \( M \) mass are \( F_T \) (the tension in the string connecting it to the \( m \) mass) and \( F_w \) (the weight of the block).

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In order to stop the tray from moving, the net force on both the \( M \) mass and the mass \( m \) loaded on the tray must be zero. Newton’s First Law applied to the \( M \) mass yields

\[
F_{w2} - F_T = 0 \Rightarrow F_T = F_{w2} = Mg
\]

Newton’s First Law applied to the \( m \) yields

\[
F_T - F_g - F_f = 0 \Rightarrow F_T - mg \sin(\theta) - \mu_s mg \cos(\theta) = 0
\]

Substituting \( F_T = Mg \), you get

\[
Mg - mg \sin\theta - \mu_s mg \cos\theta = 0 \\
mg \sin\theta + \mu_s mg \cos\theta = Mg \\
m = \frac{Mg}{g \sin\theta + \mu_s g \cos\theta}
\]

(b) As the mass is trying to slide down the incline, the force of friction is acting in the opposite direction as in (a). There is no change to the \( M \) mass, so Newton’s First Law applied to the \( M \) mass still yields

\[
F_{w2} - F_T = 0 \Rightarrow F_T = F_{w2} = Mg
\]

Newton’s First Law applied to the \( m \) yields

\[
F_T + F_g - F_f = 0 \Rightarrow F_T + mg \sin\theta - \mu_s mg \cos\theta = 0
\]

Substituting \( F_T = Mg \), you get

\[
Mg + mg \sin\theta - \mu_s mg \cos\theta = 0 \\
mg \sin\theta - \mu_s mg \cos\theta = Mg
\]
In order for the massless tray to start sliding down the inclined plane, \( m \) has to be greater than \( \frac{Mg}{g \sin \theta + \mu \cdot g \cos \theta} \).

(c) Newton’s First Law applied to the \( M \) mass yields

\[
F_T - F_{w2} = Ma \Rightarrow F_T = F_{w2} + Ma \Rightarrow F_T = Mg + Ma
\]

Newton’s First Law applied to the \( m \) yields

\[
F_g - F_f - F_T = ma \Rightarrow mg \sin \theta - \mu \cdot mg \cos \theta - F_T = ma
\]

Substituting \( F_T = Mg + Ma \), you get

\[
mg \sin \theta - \mu \cdot mg \cos \theta - Mg - Ma = ma
\]

\[
a = \frac{mg \sin \theta - \mu \cdot mg \cos \theta - Mg}{m + M} = \frac{m \sin \theta - \mu \cdot m \cos \theta - M}{m + M}
\]

Plugging in \( m = 4M \), \( \theta = 45^\circ \), and \( \mu_k = 0.3 \), you get

\[
a = \frac{(4M) \sin 45^\circ - 0.3(4M) \cos 45^\circ - M}{4M + M} = \frac{4 \sin 45^\circ - 0.3(4) \cos 45^\circ - 1}{5} = 0.2 \text{ m/s}^2
\]

5. (a) First, calculate the amount of time it takes for the ball to travel 20 m. Apply the first of the horizontal motion questions:

\[
\Delta x = v_{0x}t \Rightarrow t = \frac{\Delta x}{v_{0x}} = \frac{20 \text{ m}}{16 \cos 30^\circ \text{ m/s}} = 1.44 \text{ s}
\]

Next, calculate the height of the ball at \( t = 2.3 \text{ s} \). Apply Big Five #3 to vertical motion:

\[
\Delta y = v_{0y}t - \frac{1}{2}gt^2 = (16 \text{ m/s})\sin 30^\circ (1.44 \text{ s}) - \frac{1}{2}(10 \text{ m/s}^2)(1.44 \text{ s})^2 = 1.55 \text{ m}
\]

The student must swing the bat at a height of 1.15 m to hit the ball.

(b) The horizontal velocity of the ball is constant:

\[
v_x = v_{0x} = v_0 \cos \theta = 16 \cos 30^\circ = 13.9 \text{ m/s}
\]

As impact occurs when \( t = 1.44 \text{ s} \), the vertical velocity of the ball can be calculated using Big Five #2 to vertical motion:

\[
v_y = v_{0y} - gt = 16 \sin 30^\circ - (10 \text{ m/s}^2)(1.44 \text{ s}) = -6.4 \text{ m/s}
\]
Use the Pythagorean Theorem to solve for the magnitude of the overall velocity:

\[ v^2 = v_x^2 + v_y^2 \Rightarrow v = \sqrt{v_x^2 + v_y^2} = \sqrt{(13.9 \text{ m/s})^2 + (-6.4 \text{ m/s})^2} = 15.3 \text{ m/s} \]

(c) The impulse is equal to the change in momentum of the ball. You can calculate the horizontal and vertical components of the impulse separately:

\[ J_x = \Delta p_x = m\Delta v_x = m(v_{x,f} - v_{x,i}) = 2 \text{ kg}(-12 \text{ m/s} - 13.9 \text{ m/s}) = -51.8 \text{ N}\cdot\text{s} \]

\[ J_y = \Delta p_y = m\Delta v_y = m(v_{y,f} - v_{y,i}) = 2 \text{ kg}(5 \text{ m/s} - (-6.4 \text{ m/s}) = 22.8 \text{ N}\cdot\text{s} \]

(d) You can use the equation \( \mathbf{J} = \mathbf{F}\Delta t \) to calculate the horizontal and vertical components of the average force:

\[ F_x = \frac{J_x}{\Delta t} = \frac{-51.8 \text{ N} \cdot \text{s}}{0.05 \text{ s}} = -1,360 \text{ N} \]

\[ F_y = \frac{J_y}{\Delta t} = \frac{22.8 \text{ N} \cdot \text{s}}{0.05 \text{ s}} = 456 \text{ N} \]

Use the Pythagorean Theorem to calculate the magnitude of the average force experienced by the ball during impact:

\[ F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-1,360 \text{ N})^2 + (456 \text{ N})^2} = 1,434 \text{ N} \]

(e) The direction of the average force can be calculated using

\[ \tan(\theta) = \frac{F_y}{F_x} \Rightarrow \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{456 \text{ N}}{-1,360 \text{ N}}\right) = 18.5^\circ \]

The direction of the force is 18.5° north of west.